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ABSTRACT

The process of applying Bloom's Taxonomy of Educational Objectives to developmental math classes is described in this paper. The first section provides background and identifies the six steps in the cognitive domain, arranged from the lowest to the most complex level of learning, as knowledge, comprehension, application, analysis, synthesis, and evaluation. Next, the advantages of applying the taxonomy to developmental math courses are outlined, in terms of its role in aiding students to overcome math anxiety and in providing a structure within which the instructor can articulate the behavioral objectives for the students. Then, the use of the taxonomy for lesson planning and creative teaching is discussed, using as an example a lesson involving quadratic equations. First, the formula is stated and the students are asked to memorize it. Second, the students relate the general formula to specific applications. Third, students are asked to solve problems using the equation. Fourth, students apply the skills learned to solving word problems and relating quadratic equations to other types of equations. Fifth, students are taught to synthesize the knowledge of quadratic equations attained and other algebraic techniques. Finally, methods of problem solving are evaluated on the basis of their validity, precision, and elegance. Appendices provide additional lesson plans, applying Bloom's taxonomy to the study of simple set notation, right triangles solving, and curve sketching. (HB)

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USING BLOOM'S TAXONOMY FOR PRECISION PLANNING AND CREATIVE TEACHING IN THE DEVELOPMENTAL MATH CLASSROOM

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In the developmental math classroom, the instructor as well as the student may feel "math anxiety." The instructor's anxiety often has to do with preparing well-designed math lessons. This anxiety can be alleviated by the instructor's using Bloom's Taxonomy to help with lesson planning.

Before looking at a method of applying this Taxonomy directly to math lessons, let's consider its background.

Reviewing the History of the Taxonomy

In 1948 when Benjamin Bloom and fellow college examiners formed the idea for the now famous educational classification system, little did they realize how profound an impact they would make on the entire educational process. What has finally come to be known as Bloom's Taxonomy of Educational Objectives is a system for stating educational objectives that requires a plan of purposeful specific activities, reduces vagueness, provides a set of common terms, and improves communication among educators.

Formerly educators spoke of student mastery of subject skills in vague terms such as the following: I want my students to "really understand," "grasp the core or essence," and singly, "comprehend" (Bloom, p. 1). In speaking in such abstruse terms, curricula and test designers could not be sure whether teachers meant that students

should be able to recall the bare facts or whether students were expected to be able to evaluate the basic principles upon which the learning was based. Bloom and his colleagues decided that it was necessary to articulate precise levels of mastery from the least complex form of learning, "knowledge," to the most complex, "evaluation."

According to Bloom, the establishment of such a taxonomy, in which "the process of thinking about educational objectives, defining them, and relating them to teaching and testing procedures" could become an extremely valuable tool for educators (Bloom, p. 5).

The final result was a classification system consisting of six steps and three domains. The six steps, arranged in hierarchal order, (from the lowest to the most complex level of learning), consisted of the following in the cognitive domain: 1. Knowledge, 2. Comprehension, 3. Application, 4. Analysis, 5. Synthesis, and 6. Evaluation.

Using the taxonomy, educators could intelligently design course curricula and follow through with tests measuring these outcomes because the specific level of mastery desired (the stated behavioral objective) could be articulated. Although originally designed to aid communication among test makers, the taxonomy could serve as a resource for both classroom teachers and curricula and test designers.

Applying the Taxonomy to Math

In effect, Bloom's Taxonomy not only offers clarity in communicating objectives but also precision in lesson planning and intel-

lectual development--especially appropriate in the remedial/developmental math classroom. For if any group of students needs a carefully designed structure of progressively difficult learning activities, the developmental student needs it. Usually such students have an intense fear of math based on feelings of incompetence in basic computation, understanding math word problems, or in general, anything having to do with mathematics. Increasingly math instructors are becoming aware of such problems regarding their students, and the term math anxiety is being used to describe such mental blocks. According to Crawford, math anxiety is the manifestation of "the frustration, confusion and tension often associated with attempting math. . . a severe and pervading problem" (p. 8). In addition to the anxieties he suffers in math, the developmental math student has anxieties associated with his being able to gain access into regular college credit math classes. To be a part of the mainstream is often one of his major goals. Therefore the instructor's utilization of Bloom's taxonomy would assist this student in gaining confidence and would aid him in progressively reducing his feeling of anxiety about math. In addition, the taxonomy provides the structure for the instructor of developmental math to articulate with precision the behavioral objectives he wants exhibited by the classroom activities he wants to use in order to achieve these objectives.

Because the emphasis in this article is based on lesson planning and classroom activities for cognitive or skills development, the descriptions of the affective and psychomotor domains have been omitted.

Using the Taxonomy for Lesson Planning and Creative Teaching

The instructor of mathematics can utilize a specially designed chart (See Table 1) for planning various lessons within a given topic. This form would also provide for a description of the specific lesson activities accompanying each lesson.

For example, the instructor might begin a lesson on the quadratic formula by asking the class what it means to solve an equation and how one goes about checking answers. The quadratic formula is stated and the students are asked to memorize it. This represents the lowest level of learning involving only knowledge of the basic facts.

For the next step, Comprehension, the class would come to an understanding of the correspondence between the general quadratic equation in standard form, $a x^2 + bx + c = 0$, and specific examples of quadratic equations provided by the instructor. A lecture-discussion format might be used here.

The instructor could provide further insight into the quadratic formula by deriving it from standard form by completing the square. The instructor would facilitate discussion after each step of the derivation by asking the class to provide the justification.

After this, the class could be split into small groups of three or four students. The instructor would then give the class a quadratic equation to solve and students would work individually within their group. After a few minutes, papers are exchanged within the group, the correct answer is given and students are allowed to discuss and critique their solutions.

TABLE 1

TOPIC: Solving Quadratic Equations Using the Quadratic Formula

1. Knowledge. Knows the meaning of solving an equation. Knows the statement of the quadratic formula. Knows how to check an answer.	Instructor leads class discussion on the meaning of solving an equation and checking answers. The quadratic formula is stated.
2. Comprehension. Understands quadratic formula applies only to equations in quadratic form: $ax^2 + bx + c = 0$ Properly identifies the coefficients a, b and c in a given problem. Can justify the quadratic formula by solving $ax^2 + bc + c = 0$ by completing the square.	Class discusses correspondence of general form $ax^2 + bx + c = 0$ to specific examples of quadratic equations. Instructor derives the quadratic formula allowing students to supply justifications at each step. Class is split into small groups of 3 or 4 students. Instructor gives class a problem. Students work individually, exchange papers within their group and critique solutions.
3. Application. Can apply this method of solving quadratic equations to word problems. Can use quadratic formula to determine x intercepts of equation of the form $y = ax^2 + bx + c$. Can use the quadratic formula to solve other types of algebraic equations i.e., rational equations, radical equations, etc.	Each small group is given a word problem to work collectively. Each group sends a representative to the board to present their problem with complete explanation. Instructor illustrates other equations that give use to quadratic equations. Students work radical equations and rational equations on a handout.
4. Analysis. Recognizes necessity of standard form. Recognizes the relationship between the degree of the polynomial and the number of roots of the polynomial equation. Recognizes the relationship between the roots and the factors. Predicts the nature of the roots. Analyzes a word problem that gives rise to a quadratic equation. Analyzes a given quadratic equation to determine the most direct method of solution: factoring, completing the square or formula. Analyzes an equation in quadratic form in order to apply the quadratic formula.	During instructor-led discussion key relationships among powers, roots and factors are examined. Students in small groups analyze word problems to construct the quadratic equation and determine the most efficient and effective means of solution.
5. Synthesis. Use quadratic formula to solve other equations in quadratic form such as $2\sin^2\theta - \sin\theta - 1 = 0$ $\left(\frac{x}{x+1}\right)^2 + 5\left(\frac{x}{x+1}\right) + 4 = 0$ $x^4 + 2x^2 + 3 = 0$, etc. Suggests ways to use the quadratic formula in systems of non linear equations.	Each small group is given a different problem involving an equation in quadratic formula. After an instructor-led discussion, involving some examples, each group works collectively on its problem. A spokesman from the group presents the problem on the board with explanation.
6. Evaluation. Judges the validity of a method and appropriateness of solutions when using quadratic formula to solve word problems. Can interpret answer in light of the problem. Can compare and contrast the value of different methods of solution for a given quadratic equation. Can estimate and interpret irrational solutions. Can interpret imaginary solutions. Can discriminate between solutions and extraneous solutions.	Class and instructor critiques each group presentation (word problems and general quadratic forms) according to validity and elegance of method, appropriateness of answer, and interpretation of results and information.

After the students have had sufficient practice solving specific quadratic equations, they are ready to apply the knowledge and skills in solving word problems. The instructor would give each group a different word problem involving a quadratic equation. Members of the group would work collectively on their problem and a spokesman for the group would be chosen to present the problem to the class. Word problems might involve applications of quadratic equations to areas of geometric figures, paths of projectiles, etc.

Also, in an instructor-led discussion, the teacher may show how this technique for solving quadratic equations could be useful in solving other types of equations (rational equations, radical equations) that give rise to quadratic equations. Additional work could be given on a handout which would allow students to solve these types of equations individually.

As the instructor develops the quadratic formula and shows its many applications, he encourages analytic thinking by pointing out certain key relationships among the powers, factors and solutions. For example, if a quadratic equation factors, it has at most two distinct factors, hence it may have no more than two distinct solutions. Analyzing details in this manner, the students are led to their first glimpse of the Factor Theorem, i.e. $x=a$ is a root of $P(x)=0$ if and only if $(x-a)$ is a factor of $P(x)$. Also relationships among the sum and difference of roots and the coefficients of the quadratic equation might be examined.

Furthermore, in the small group activity of solving word problems, a good deal of analytical thinking is occurring in utilizing the information to construct an equation. Students may have at their disposal other methods for solving a quadratic equation. They must analyze each equation to determine the best approach to use.

Frequently, students complain that they forget previous math lessons in their anxiety to comprehend the new lessons. Synthesis, the fifth category of the taxonomy, allows the instructor to integrate newly learned techniques with old skills. Having mastered the basic techniques of solving quadratic equations using the quadratic formula, examples might be generalized to include other equations in quadratic form. After an instructor-led discussion involving a few examples, each small group would again be given a problem to solve. Each question would give students the opportunity to synthesize previously learned algebraic techniques with the quadratic formula to equations in quadratic form. A spokesman form each group presents the problem at the board with an explanation.

The sixth step, Evaluation, is used upon completion of each problem. Methods of solution will be examined and judged on the basis of their validity and elegance and answers on their appropriatness. Mathematical precision will be stressed. Students will be asked to interpret their answers in light of the questions and disregard extraneous solutions. Hence, evaluation will occur on a personal level and in class discussion at the conclusion of small group activities.

In Summation:

Bloom's Taxonomy can be used as a guide for precision in lesson planning and for creative, classroom activity in that it enables instructors to move the developmental math student from the basic level of learning (Knowledge) to more complex intellectual tasks (Analysis, Synthesis, and Evaluation). In that each step of the taxonomy precisely describes the behavioral objectives of that step and provides a base of learning for the next higher-level step, instructors can use the taxonomy in planning their sequence of lessons and the student-directed activities as board work, student-to-student recitation and discussion, small group work, and study-buddy sessions, all offer opportunities for students to apply what they have learned on each of the six steps of the taxonomy. The productive experience gained by students through the careful, comprehensive planning suggested by Bloom's method, offers hope of swifter transit to those struggling to enter the educational mainstream.

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Addenda

Additional lessons that have been developed using Bloom's Taxonomy as a guide.

TABLE 1

MAJOR CATEGORIES IN THE COGNITIVE DOMAIN OF THE TAXONOMY OF EDUCATIONAL OBJECTIVES (BLOOM, 1956)

Descriptions of the Major Categories in the Cognitive Domain	*Illustrative General Instructional Objectives*	*Illustrative Behavioral Terms for Stating Specific Learning Outcomes*
Topic: Simple Set Notation		
1. Knowledge. Knowledge is defined as the remembering of previously learned material. This may involve the recall of a wide range of material, from specific facts to complete theories, but all that is required is the bringing to mind of the appropriate information. Knowledge represents the lowest level of learning outcomes in the cognitive domain.	Knows common terms Knows specific facts Knows methods and procedures Knows basic concepts Knows principles	Defines, describes, identifies, labels, lists, matches, names, outlines, reproduces, selects, states
Lesson Activity (Behavioral Objectives) Knows definition and use of set notation. Knows that commas and braces denotes set notation. Knows how to transform a list of items into set notation. Knows how to identify the number of elements in a set. Knows that repetition in a set doesn't count.	Student-Directed Activity Discuss examples of items in lists. Select the appropriate groups of items for specific sets. Discuss why certain items could not be selected for specific groups.	
2. Comprehension. Comprehension is defined as the ability to grasp the meaning of material. This may be shown by translating material from one form to another (words to numbers), by interpreting material (explaining or summarizing), and by estimating future trends (predicting consequences or effects). These learning outcomes go one step beyond the simple remembering of material, and represent the lowest level of understanding.	Understands facts and principles Interprets verbal material Interprets charts and graphs Translates verbal material for mathematical formulas Estimates future consequences implied in data Justifies methods and procedures	Converts, defends, distinguishes, estimates, explains, extends, generalizes gives examples, infers, paraphrases, predicts, rewrites, summarizes
Lesson Activity (Behavioral Objectives) Knows how to interpret verbal material to set notation. Distinguishes between "finite" and "infinite" sets Interprets the type of set and explains the appropriateness of the elements therein. Has thorough understanding of the basic facts and principles of set notation.	Student-Directed Activity Individual work from worksheet of verbal instructions for set notation. Student volunteers place work on blackboard; then each one explains his examples of set notation, discussing type of set, number of elements, finite and infinite. Student mistakes are corrected by fellow students	
3. Application. Application refers to the ability to use learned material in new and concrete situations. This may include the application of such things as rules, methods, concepts, principles, laws, and theories. Learning outcomes in this area require a higher level of understanding than those under comprehension.	Applies concepts and principles to new situations Applies laws and theories to practical situations Solves mathematical problems Constructs charts and graphs Demonstrates correct usage of a method or procedure	Changes, computes, demonstrates, discovers, manipulates, modifies, operates, predicts, prepares, produces, relates, shows, solves, uses
Lesson Activity (Behavioral Objectives) Knows how to apply principles of set notation to new situations.	Student-Directed Activity Applies set notation to - supermarket areas (areas not discussed in first lesson; broad topics from other subject areas (famous artists, historical places, types of plants, etc.	

Table II

MAJOR CATEGORIES IN THE COGNITIVE DOMAIN OF THE TAXONOMY OF EDUCATIONAL OBJECTIVES (BLOOM, 1956)

Descriptions of the Major Categories in the Cognitive Domain	*Illustrative General Instructional Objectives*	*Illustrative Behavioral Terms for Stating Specific Learning Outcomes*
4. **Analysis.** Analysis refers to the ability to break down material into its component parts so that its organizational structure may be understood. This may include the identification of the parts, analysis of the relationships between parts, and recognition of the organizational principles involved. Learning outcomes here represent a higher intellectual level than comprehension and application because they require an understanding of both the content and the structural form of the material.	Recognizes unstated assumptions Recognizes logical fallacies in reasoning Distinguishes between facts and inferences Evaluates the relevancy of data Analyzes the organizational structure of a work (art, music, writing)	Breaks down, diagrams, differentiates, discriminates, distinguishes, identifies, illustrates, infers, outlines, points out, relates, selects, separates, subdivides.
Lesson Activity (Behavioral Objectives) Analyzes elements of various sets to determine which sets can be designated as sub-sets.	Student-Directed Activity Small group work - Each group analyzes sets to determine which ones are subsets. Each group chairman then discusses group consensus with class.	
5. **Synthesis.** Synthesis refers to the ability to put parts together to form a new whole. This may involve the production of a unique communication (theme or speech), a plan of operations (research proposal), or a set of abstract relations (scheme for classifying information). Learning outcomes in this area stress creative behaviors, with major emphasis on the formulation of new patterns or structures.	Writes a well organized theme Gives a well organized speech Writes a creative short story (or poem, or music) Proposes a plan for an experiment Integrates learning from different areas into a plan for solving a problem Formulates a new scheme for classifying objects (or events, or ideas)	Categorizes, combines, compiles, composes, creates, devises, designs, explains, generates, modifies, organizes, plans, rearranges, reconstructs, relates, reorganizes, revises, rewrites, summarizes, tells, writes
Lesson Activity (Behavioral Objectives **Formulates** new patterns of sets using previously learned material, the number system.	Student-Directed Activity Study-buddy system (2 students) - Both review the material on the number system. As a team, they work on designing sets involving natural numbers, integers, and certain sequences in one rational number system. The work is then exchanged with another team for the next level, evaluation.	
6. **Evaluation.** Evaluation is concerned with the ability to judge the value of material (statement, novel, poem, research report) for a given purpose. The judgments are to be based on definite criteria. These may be internal criteria (organization) or external criteria (relevance to the purpose) and the student may determine the criteria or be given them. Learning outcomes in this area are the highest in the cognitive hierarchy because they contain elements of all of the other categories, plus conscious value judgments based on clearly defined criteria.	Judges the logical consistency of written material Judges the adequacy with which conclusions are supported by data Judges the value of a work (art, music, writing) by use of internal criteria Judges the value of a work (art, music, writing) by use of external standards of excellence	Appraises, compares, concludes, contrasts, criticizes, describes, discriminates, explains, justifies, interprets, relates, summarizes, supports
Lesson Activity (Behavioral Objectives) **Judges** material based on the criteria established.	Student-Directed Activity Study-buddy teams evaluate work done by the other team. Evaluating teams must support all judgments by citing the criteria pertinent to their judgment.	

*Tables are a composite of those found in N. E. Gronlund, *Stating Behavioral Objectives for Classroom Instruction* (New York: Macmillan, 1970)

TABLE 1

MAJOR CATEGORIES IN THE COGNITIVE DOMAIN OF THE TAXONOMY OF EDUCATIONAL OBJECTIVES (BLOOM, 1956)

Descriptions of the Major Categories in the Cognitive Domain	*Illustrative General Instructional Objectives*	*Illustrative Behavioral Terms for Stating Specific Learning Outcomes*
Topic: Right Triangles Solving		
1. **Knowledge.** Knowledge is defined as the remembering of previously learned material. This may involve the recall of a wide range of material, from specific facts to complete theories, but all that is required is the bringing to mind of the appropriate information. Knowledge represents the lowest level of learning outcomes in the cognitive domain.	Knows common terms Knows specific facts Knows methods and procedures Knows basic concepts Knows principles	Defines, describes, identifies, labels, lists, matches, names, outlines, reproduces, selects, states
Lesson Activity (Behavioral Objectives) Knows the definition of a right triangle. Knows the statement of the Pythagorean Theorem. Knows that the sum of the angles in a right triangle is 180^{C}. Knows the right triangle definitions of the six trigonometric functions. Knows that to solve a right triangle means to determine all sides and angles.	Student-Directed Activity In classroom discussion definitions and facts are presented and examined. Examples are given.	
2. Comprehension. Can construct a right triangle and label its sides and angles to fit a given verbal description. Comprehends that the Pythagorean Theorem is valid only if the triangle contains a right angle. Comprehends that the words "opposite" and "adjacent" are relative to the angle under consideration. Can judge when the given information is sufficient to determine a specific side or angle	Examples are put on the board by the instructor illustrating the various possibilities. Special care is taken discussing word problems. Students are given examples to set up individually at their desks.	
3. Application. Can apply methods of solving right triangles to word problems. Can apply methods of solving right triangles to problems in engineering, physics, architecture, drafting, surveying, etc.	A handout illustrating applications of right triangles to other fields is given. Individuals present problems at the board with complete explanations and interpretations.	
4. Analysis. Given certain sides and angles, the student can determine which of the six trigonometric functions would be appropriate to use to determine a specific side or angle.	Students are paired together to form learning dyads. Each student creates a problem for the other to solve involving certain sides and angles. Students discuss why certain approaches are preferred to others.	
5. Synthesis. Can use methods of solving right triangles to determine the altitude in any triangle given appropriate information. Can subdivide general triangles into right triangles and apply methods to determine lengths and angles.	Class is broken down into small groups of 3 or 4 students. Each group is given a different problem that involves solving a right triangle such as determining the altitude of <u>any</u> triangle given certain information. Each group chooses a spokesperson to present their problem on the board with a complete explanation.	
6. Evaluation. Judges solution of problem based on criteria that includes checking that the sum of any pair of sides exceeds the third, sum of angles is 180^{o} and compare size of angles and corresponding sides. Can appreciate elegant and direct solutions.	Entire class (including instructor) evaluates work done by each small group. Judgments are supported by citing criteria pertinent to judgment.	

TABLE II

TOPIC: Exponential Growth and Decay.

1. Knowledge. Knowledge that an instantaneous rate of change is a derivative. Knows how to solve a simple differential equation by separating variables. Knows how to integrate $\frac{dy}{y} = k\,dt$. Knows that $f(x) = e^x$ and $g(x) = \ln x$ are inverse functions and that $(f \circ g)(x) = x$ and $(g \circ f)(x) = x$. Knows that $e^0 = 1$. Knows how to solve a simple exponential equation of the form $a = e^x$	Discuss the idea of instantaneous rate of change. Review how to integrate $\int \frac{dy}{y}$ and $\int K\,dt$. Discuss what it means to solve a differential equation. Reviews basic algebraic properties of exponentials and logarithms.
2. Comprehension. In a word problem, the student can identify pertinent information i.e., initial amount present, etc. Can substitute information in a word problem into the form $y(t) = C \cdot e^{kt}$ in order to determine unknowns. Can derive the form $y(t) = C\,e^{kt}$ from $y' = ky$ and explain why $y' = ky$ accurately describes an exponential growth or decay problem.	Instructor led discussion at the board stating the general problem, and the differential equation used in the model. The instructor derives the general relationship $y = Ce^{kt}$ asking students to provide the justification for each step. Students are asked to identify and interpret information given in an example (a word problem).
3. Application. Knows how to apply principles of exponential growth and decay to radioactive decay problems in chemistry, growth of bacteria problems in biology, population models and interest problems or any situation where the rate of change depends directly on amount present.	Class is split into small groups of 3 or 4 students. Each group is given a growth or decay problem from a different field to solve. Each group sends a spokesman to the board to present the problem and explain procedure.
4. Analysis. Can analyze a word problem or real life situation involving exponential growth or decay to determine the appropriateness of $y(t) = C\,e^{kt}$ as a description of the phenomena. Can isolate and utilize given information to solve problem.	Each group of 3 or 4 students carefully analyzes the word problem they are given, utilizing the given information to solve the problem.
5. Synthesis. Within the derivation of $y(t) = C\,e^{kt}$ from $y' = ky$ the student will make use of previously learned analytical and algebraic techniques such as integrating $\frac{dy}{y}$ and $k\,dt$, solving for k, in an equation of the form $a = e^k$ and using properties of the natural logarithm function when necessary.	Students can reconstruct the derivation of $y = Ce^{kt}$ from $y' = ky$ using previously learned analytic and algebraic techniques. In actual problem solving, students construct solution utilizing algebraic properties of logarithms and exponentials.
6. Evaluation. Judges solution to growth or decay problem as reasonable and a fair description of reality, sees the value of logarithmic and exponential functions to real life application. Appreciates the power, elegance and accuracy of the solution.	Class and instructor evaluate each small group's presentation according to the criteria established.

TABLE III

TOPIC: Solving Quadratic Equations Using the Quadratic Formula

1. Knowledge. Knows the meaning of solving an equation. Knows the statement of the quadratic formula. Knows how to check an answer.	Instructor leads class discussion on the meaning of solving an equation and checking answers. The quadratic formula is stated.
2. Comprehension. Understands quadratic formula applies only to equations in quadratic form: $ax^2 + bx + c = 0$ Properly identifies the coefficients a, b and c in a given problem. Can justify the quadratic formula by solving $ax^2 + bc + c = 0$ by completing the square.	Class discusses correspondence of general form $ax^2 + bx + c = 0$ to specific examples of quadratic equations. Instructor derives the quadratic formula allowing students to supply justifications at each step. Class is split into small groups of 3 or 4 students. Instructor gives class a problem. Students work individually, exchange papers within their group and critique solutions.
3. Application. Can apply this method of solving quadratic equations to word problems. Can use quadratic formula to determine x intercepts of equation of the form $y = ax^2 + bx + c$. Can use the quadratic formula to solve other types of algebraic equations i.e., rational equations, radical equations, etc.	Each small group is given a word problem to work collectively. Each group sends a representative to the board to present their problem with complete explanation. Instructor illustrates other equations that give use to quadratic equations. Students work radical equations and rational equations on a handout.
4. Analysis. Recognizes necessity of standard form. Recognizes the relationship between the degree of the polynomial and the number of roots of the polynomial equation. Recognizes the relationship between the roots and the factors. Predicts the nature of the roots. Analyzes a word problem that gives rise to a quadratic equation. Analyzes a given quadratic equation to determine the most direct method of solution: factoring, completing the square or formula. Analyzes an equation in quadratic form in order to apply the quadratic formula.	During instructor-led discussion key relationships among powers, roots and factors are examined. Students in small groups analyze word problems to construct the quadratic equation and determine the most efficient and effective means of solution.
5. Synthesis. Use quadratic formula to solve other equations in quadratic form such as $2\sin^2\theta - \sin\theta - 1 = 0$ $\left(\frac{x}{x+1}\right)^2 + 5\left(\frac{x}{x+1}\right) + 4 = 0$ $x^4 + 2x^2 + 3 = 0$, etc. Suggests ways to use the quadratic formula in systems of non linear equations.	Each small group is given a different problem involving an equation in quadratic formula. After an instructor-led discussion, involving some examples, each group works collectively on its problem. A spokesman from the group presents the problem on the board with explanation.
6. Evaluation. Judges the validity of a method and appropriateness of solutions when using quadratic formula to solve word problems. Can interpret answer in light of the problem. Can compare and contrast the value of different methods of solution for a given quadratic equation. Can estimate and interpret irrational solutions. Can interpret imaginary solutions. Can discriminate between solutions and extraneous solutions.	Class and instructor critiques each group presentation (word problems and general quadratic forms) according to validity and elegance of method, appropriateness of answer, and interpretation of results and information.

TABLE IV

TOPIC: Curve Sketching

1. Knowledge. Know definition of function, domain, range, intercepts, relative extrema, points of inflection, increasing function, decreasing function, concave upward, concave downward and critical numbers. Knows that division by zero is undefined. Knows how to find x and y intercepts, how to test for symmetry and how to determine vertical and horizontal asymptotes. Knows how to differentiate algebraic functions. Knows the statement of the first derivative test and the second derivative test. Knows relationships between sign of derivatives on an inverval and whether it is increasing decreasing, concave upward or concave downward.	In an instructor-led discussion, the instructor presents all of the main definitions, theorems and procedures.
2. Comprehension. Can draw an increasing (decreasing) function. In a given graph, can identify intervals on which f is increasing or decreasing. Can draw a function that is concave upward or concave downward. In a given graph, can identify intervals on which f is concave upward or concave downward. Can identify on graphs points where $f' = 0$ and f' does not exist.	Students work individually on a set of problems that involve sketching arcs that are increasing, decreasing, concave upward or concave downward. Students are given graphs for which they identify intervals where f is increasing, decreasing, concave upward, concave downward, $f' = 0$ and f' fails to exist.
3. Application. Given any algebraic function, sketch the graph using the techniques of calculus. Curve sketching may be encountered in the context of more involved problems in chemistry, biology, physics, psychology, economics, etc.	The class is broken into small groups of 3 or 4 students. Each group is given a different problem relating curve sketching to a discipline i.e., in chemistry, sketch the graph of a curve representing a reaction where points of inflection correspond to equivalence points of the reaction; in physics, sketch the trajectory of a projectile fired directly upward where the relative maximum value corresponds to the greatest height achieved. Each group elects a spokesman to present their problem to the class with a complete explanation.
4. Analysis. Analyzes a given function and its derivatives to determine intercepts, asymptotes, symmetry, extrema, points of inflection, intervals of increase or decrease and concavity.	Each small group is given a curve to sketch. f and its first and second derivatives are analyzed to determine critical features of the graph.
5. Synthesis. Organizes all results from the above-mentioned analysis to incorporate into a sketch of f consistent with all details. Incorporates basic algebraic techniques to solve equations giving intercepts, asymptotes and critical numbers. Uses algebraic techniques to simplify derivatives.	After each group analyzes its function, all of the details are organized into a chart from which f may be sketched. The spokesman from each group presents the problem before the class providing insights into how the disparate and disjoint information gotten from analyzing f, f' and f'' was organized and related to generate the graph of f
6. Evaluation. Judges sketch of f against typical features of similar graphs of polynomials, rational functions, conic sections, etc. noting number of intercepts, turning points, asymptotes, domain restrictions, basic shape, etc.	Individual group members evaluate sketch before presentation to class based on criteria cited. Class and instructor critique the sketch of f based on criteria cited.

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